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Convection driven by concentration- and temperature-dependent surface tension

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This paper generalizes the linear stability analysis of Pearson for Marangoni instability to the case where surface tension depends on both temperature and solute concentration. The results are expressed in terms of a thermal Marangoni number B_T and a solutal Marangoni number B_S. It is found when $B_S > 0$ the onset of instability has the form of stationary convection, while when $B_S < 0$ there are circumstances in which the onset of instability is in the form of oscillatory convection.

1. Introduction

In this paper the onset of convection, induced by surface tension, in a horizontal layer of fluid is examined. In addition to heating the fluid from below or above, variations in solute concentration are considered. As in the case of Bénard cells induced by density variations, certain minimum requirements must be satisfied in order that cells may develop under the action of surface tension forces. Associated with these requirements are the dimensionless Marangoni and solutal Marangoni numbers which take critical values at the onset of convection. For the situation in which solute concentration is constant the reader is referred to the works of Pearson (1958); Davis (1969*a*) and Vidal & Acrivos (1966).

In the analysis presented here buoyancy terms are neglected in the equations of motion (cf. Pearson 1958). This is equivalent to considering a zero-gravity environment, and enables the effects of surface tension to be more clearly seen.

Attention is restricted to the linearized equations, and in the first part the critical Marangoni numbers for the onset of stationary convection are obtained. The analysis is then extended to examine the possibility of oscillatory convective instability occurring.

One of the main applications of this work is in the area of crystal growing in a low-gravity environment, e.g. an orbiting spacecraft. In crystals grown from melts and aqueous solution, convective flow is in general a beneficial influence, since it serves to reduce the diffusional barrier to crystal growth. However, it has been found experimentally that when the flow has an oscillatory component there are marked impurity striations in the crystals. In his review of hydrodynamics in crystal growth, Hurle (1976) has pointed out the need for further detailed study of convection in near-zero gravity. In space processing, when buoyancy-driven convection is negligible, it has been hoped that the inhomogeneous solute distribution in crystals, caused on Earth by buoyancy-driven time-dependent convection, could be avoided. However, in this situation surface tension driven convection plays a dominant role. The aim of this paper is to outline a procedure for determining, for a given material, whether oscillatory or stationary convection will be preferred.

It is found that when both the Marangoni and solutal Marangoni numbers are positive (i.e. when both temperature and concentration effects are destabilizing) then oscillatory instability does not occur and the principle of the exchange of stabilities is valid. However, when the Marangoni numbers have opposite signs, oscillatory instability occurs and values of the Prandtl and Schmidt numbers are found for which oscillatory instability is preferred to stationary convection. Calculations are performed for an aqueous solution of magnesium sulphate.

An analysis of convection in a box is also included, and there the influence that sidewalls have on the onset of convection is detailed. Critical values of the Marangoni numbers for the onset of convection are obtained. These values are the starting point in a nonlinear analysis to predict sequences of transition from one steady convection state to another (see Rosenblat, Homsy & Davis 1982).

2. The governing equations

A small-disturbance analysis is performed for the Oberbeck–Boussinesq equations (Joseph 1976) when the fluid occupies a layer of uniform thickness d whose lower surface is in contact with a fixed plane ($z = 0$) while the upper surface ($z = d$) is free. All physical properties of the fluid are assumed constant except surface tension, which varies linearly with temperature and concentration. Throughout the work, gravity effects will be neglected and the free surface will be taken to be flat.

The steady-state solution is one with both temperature and concentration gradients linear. To study the stability of this solution we consider a perturbation $(\boldsymbol{u}, \theta, c)$, where $\boldsymbol{u} = (u, v, w)$ is the perturbation velocity, θ the temperature and c the solute concentration. A normal-mode analysis with time factor σ (possibly complex) and non-dimensional wavenumber α yields the non-dimensional linearized equations:

$$(\mathrm{D}^2-\alpha^2)(\mathrm{D}^2-\alpha^2-\sigma)W = 0, \quad (\mathrm{D}^2-\alpha^2-Pr\sigma)\Theta = W, \quad (\mathrm{D}^2-\alpha^2-Sc\sigma)C = W, \tag{2.1a,b,c}$$

where W, Θ and C are functions only of z, $\mathrm{D} \equiv \mathrm{d}/\mathrm{d}z$, and Pr and Sc are the Prandtl and Schmidt numbers. Equations (2.1) must be solved subject to the boundary conditions

$$W(0) = \mathrm{D}W(0) = 0, \tag{2.2a}$$

$$\Theta(0) = 0, \quad \mathrm{D}C(0) = 0 \quad \text{or} \quad C(0) = 0, \tag{2.2b}$$

$$W(1) = 0, \quad \mathrm{D}^2W(1) = \alpha^2 B_\mathrm{T}\Theta(1) + \alpha^2 B_\mathrm{S} C(1), \tag{2.2c}$$

$$\mathrm{D}\Theta(1) + L\Theta(1) = 0, \quad \mathrm{D}C(1) + KC(1) = 0. \tag{2.2d}$$

In the above B_T and B_S represent the thermal and solutal Marangoni numbers. $-B_\mathrm{T}$ and $-B_\mathrm{S}$ are measures of the rate of change of surface tension with respect to temperature and concentration, respectively, and are also dependent on the temperature and concentration gradients. The equation in which they occur equates the change in surface tension due to temperature and concentration variations across the surface to the shear stress experienced by the fluid at the surface. Equations (2.2*d*) are the general radiation conditions, and L and K may take values between 0 and ∞. $L \rightarrow 0$ corresponds to an insulating boundary and $L \rightarrow \infty$ to a conducting boundary. On the plane $z = 0$ only the extreme values are considered (2.2*b*). (Since the two extreme values do not lead to radically different results (see §§3 and 4) the finding of an exhaustive solution for arbitrary values of L and K shall not be attempted.)

3. Stationary convection

In the problem under consideration, the underlying operator of the linear problem is not symmetric and so we are unable to deduce that σ is real. Thus the possibility of oscillatory instability cannot be excluded and will be examined in §§6–8. However, in this section we shall examine the onset of stationary convection for which the equations relevant to marginal stability are obtained by setting $\sigma = 0$ in (2.1), i.e.

$$(\mathrm{D}^2-\alpha^2)^2 W = 0, \quad (\mathrm{D}^2-\alpha^2)\Theta = W, \quad (\mathrm{D}^2-\alpha^2)C = W. \qquad (3.1\,a,b,c)$$

The solution of (3.1*a*) subject to $W(0) = W(1) = \mathrm{D}W(0) = 0$ is

$$W = A\left(\sinh \alpha z + \frac{\alpha\cosh\alpha - \sinh\alpha}{\sinh\alpha} z \sinh\alpha z - \alpha z\cosh\alpha z\right), \quad A \text{ some constant.}$$

The solution of (3.1*b*) subject to $W(0) = W(1) = \mathrm{D}W(0) = 0$ and $\mathrm{D}\Theta(1) = -L\Theta(1)$, $\Theta(0) = 0$ is

$$\Theta = A\left[\frac{3}{4\alpha} z\cosh\alpha z + \frac{\alpha\cosh\alpha-\sinh\alpha}{4\alpha\sinh\alpha} z^2\cosh\alpha z - \tfrac{1}{4}z^2\sinh\alpha z - \frac{\alpha\cosh\alpha-\sinh\alpha}{4\alpha^2\sinh\alpha} z\sinh\alpha z \right.$$
$$\left. - \left\{\frac{\alpha^2\cosh^2\alpha + \alpha\sinh\alpha\cosh\alpha + \sinh^2\alpha + L(\alpha^2+\alpha\sinh\alpha\cosh\alpha+\sinh^2\alpha)}{4\alpha^2\sinh\alpha(\alpha\cosh\alpha + L\sinh\alpha)}\right\}\sinh\alpha z\right].$$

It is easily seen that the solution of (3.1*c*) with $\mathrm{D}C(1) = -KC(1)$ and $C(0) = 0$ is the same as that for Θ with L replaced by K. Finally the solution of (3.1*c*) with $\mathrm{D}C(1) = -KC(1)$ and $\mathrm{D}C(0) = 0$ is

$$C = A\left[\frac{3}{4\alpha} z\cosh\alpha z + \frac{\alpha\cosh\alpha-\sinh\alpha}{4\alpha\sinh\alpha} z^2\cosh\alpha z - \tfrac{1}{4}z^2\sinh\alpha z - \frac{\alpha\cosh\alpha-\sinh\alpha}{4\alpha^2\sinh\alpha} z\sinh\alpha z \right.$$
$$\left. - \frac{3}{4\alpha^2}\sinh\alpha z - \left\{\frac{(\alpha\cosh\alpha-\sinh\alpha)^2 + K(\alpha^2+\alpha\sinh\alpha\cosh\alpha-2\sinh^2\alpha)}{4\alpha^2\sinh\alpha(\alpha\sinh\alpha + K\cosh\alpha)}\right\}\cosh\alpha z\right].$$

Substitution of the last boundary condition, namely

$$\mathrm{D}^2W(1) = \alpha^2 B_{\mathrm{T}}\Theta(1) + \alpha^2 B_{\mathrm{S}} C(1),$$

yields a relationship between B_{T}, B_{S}, L, K and α.

When $\Theta(0) = C(0) = 0$ (i.e. a conducting permeable base) this relationship can be expressed as

$$2\alpha(\alpha-\sinh\alpha\cosh\alpha) - \frac{\alpha^3\cosh\alpha - \sinh^3\alpha}{4(\alpha\cosh\alpha + L\sinh\alpha)}B_{\mathrm{T}} - \frac{\alpha^3\cosh\alpha - \sinh^3\alpha}{4(\alpha\cosh\alpha + K\sinh\alpha)}B_{\mathrm{S}} = 0.$$

Also $\Theta(0) = \mathrm{D}C(0) = 0$ leads to the relationship

$$2\alpha(\alpha-\sinh\alpha\cosh\alpha) - \frac{\alpha^3\cosh\alpha - \sinh^3\alpha}{4(\alpha\cosh\alpha + L\sinh\alpha)}B_{\mathrm{T}}$$
$$- \frac{\alpha^3\sinh\alpha - \alpha^2\cosh\alpha + 2\alpha\sinh\alpha - \sinh^2\alpha\cosh\alpha}{4(\alpha\sinh\alpha + K\cosh\alpha)}B_{\mathrm{S}} = 0.$$

When $B_{\mathrm{S}} = 0$ these reduce to the equation that Pearson (1958) obtained for a conducting lower boundary.

In figures 1 and 2 the marginal stability curves are plotted for various values of L and K. To obtain the curves first fix a value of B_{S} and then calculate the critical value of α that makes B_{T} a minimum. Tables 1 and 2 give values of the critical wavenumber α and Marangoni number B_{T} for $B_{\mathrm{S}} = -50$, 0, 100. Figure 3 shows a plot of α against B_{T} for $B_{\mathrm{S}} = -50$, 0, 100.

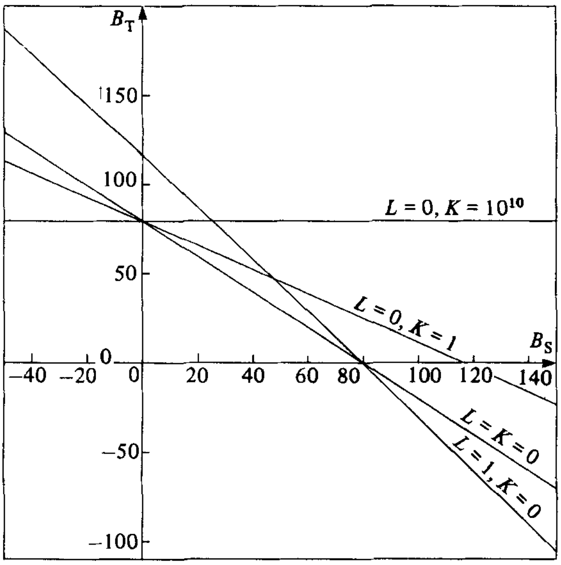

FIGURE 1. Calculated marginal stability curves, B_S versus B_T for a conducting permeable base (i.e. $\Theta(0) = C(0) = 0$).

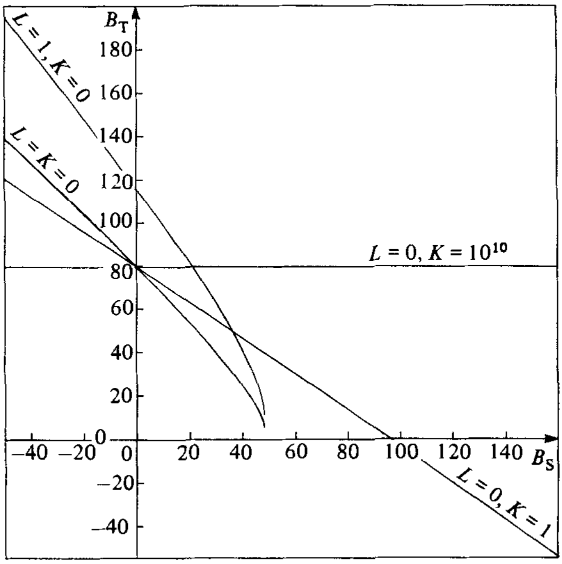

FIGURE 2. Calculated marginal stability curves, B_S versus B_T for a conducting impermeable base (i.e. $\Theta(0) = DC(0) = 0$).

The missing values in table 2 are accounted for by the fact that there is an asymptote at $B_S = 48$. As $B_S \rightarrow 48$, $\alpha \rightarrow 0$ and $B_T \rightarrow -\infty$. For $B_S > 48$ there is no effect on B_T.

4. Conclusions

First consider figures 1 and 2, which refer to a fluid with a permeable and an impermeable conducting base respectively. In both figures when $L = 0$ and $K = 10^{10}$ then B_T is constant. The thermal effect is then decoupled from the solute effect. Apart

		$B_S = -50$		$B_S = 0$		$B_S = 100$	
L	K	α	B_T	α	B_T	α	B_T
0	0	1.99	129.6	1.99	79.6	1.99	-20.4
0	1	1.89	113.1	1.99	79.6	2.21	11.2
1	0	2.40	187.2	2.25	116.1	1.93	-30.4
1	1	2.25	166.1	2.25	116.1	2.25	16.1
5	5	2.60	300.6	2.60	250.6	2.60	150.6
10	10	2.74	463.4	2.74	413.4	2.74	313.4
10^{10}	10^{10}	3.01	3.2×10^{11}	3.01	3.2×10^{11}	3.01	3.2×10^{11}
10^{10}	0	3.66	4.7×10^{11}	3.01	3.2×10^{11}	1.82	-1.0×10^{11}
0	10^{10}	1.99	79.6	1.99	79.6	1.99	79.6

TABLE 1. Conducting permeable base

		$B_S = -50$		$B_S = 0$		$B_S = 100$	
L	K	α	B_T	α	B_T	α	B_T
0	0	2.29	139.1	1.99	79.6		
0	1	2.07	120.2	1.99	79.6	1.73	-3.1
1	0	2.61	195.0	2.25	116.1		
1	1	2.41	172.7	2.25	116.1	1.72	-4.8
5	5	2.65	304.6	2.60	250.6	2.47	141.7
10	10	2.77	466.7	2.74	413.4	2.67	306.5
10^{10}	10^{10}	3.01	3.2×10^{11}	3.01	3.2×10^{11}	3.01	3.2×10^{11}
10^{10}	0	3.73	4.7×10^{11}	3.01	3.2×10^{11}		
0	10^{10}	1.99	79.6	1.99	79.6	1.99	79.6

TABLE 2. Conducting impermeable base

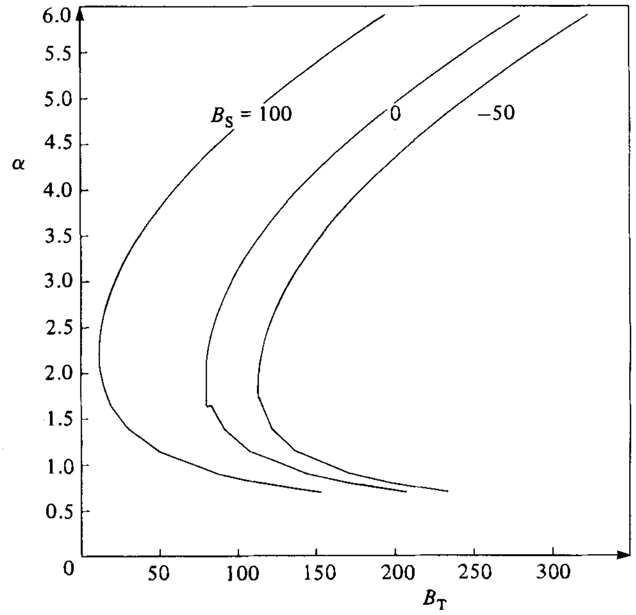

FIGURE 3. Calculated stability curves, B_T versus α, for various values of B_S. $L = 0$, $K = 1$ at the free surface and the base is conducting and permeable.

from situations where decoupling takes place, it can be seen that B_T decreases with increase of B_S. Thus the two agencies causing instability reinforce each other. With a conducting permeable base and $L = K$ then the temperature perturbation θ and the concentration perturbation c satisfy formally identical boundary conditions. Thus the stability curve in the $(B_\mathrm{T}, B_\mathrm{S})$-plane is the straight line

$$B_\mathrm{T}+B_\mathrm{S} = B_\mathrm{T}^\mathrm{c},$$

where B_T^c is the critical value of B_T when $B_\mathrm{S} = 0$. When there is a straight line there is maximum reinforcement of the two effects. Note that the coupling between B_T and B_S is less tight as L and K increase. Also larger positive values of L and K lead to greater stability.

From tables 1 and 2 the following conclusions may be drawn.

(i) As L increases (for fixed K) the corresponding wave number increases, so that the size of the convection cell decreases.

(ii) For a conducting permeable boundary and $L = K$ then the critical wavenumber is the same for all $(B_\mathrm{T}, B_\mathrm{S})$-combinations.

In addition, we note that Pearson (1958), when considering the situation where $B_\mathrm{S} = 0$, obtained the critical values $\alpha = 2.0$, $B_\mathrm{T} = 80$ for a conducting lower boundary. These correspond to the results obtained here.

In figure 3 the marginal stability curves for a conducting permeable base have been plotted. These curves display a critical (minimum) value of B_T corresponding to a particular value of α for which stable disturbances are first possible. The curve plotted for $B_\mathrm{S} = 0$ is that obtained by Pearson. It can be seen that as B_S increases the critical wave number increases.

5. Convection in a box

In this section the influence which adiabatic impenetrable sidewalls exert on the onset of convection will be investigated briefly. Suppose the fluid is in a rectangular box where for simplicity the liquid is allowed to slip on the sidewalls. B_T is found as a function of the box dimensions and B_S.

Adopting the procedure of Rosenblat *et al.* (1982), the boundary conditions to be satisfied at the sidewalls are

$$u = w_x = v_x = \theta_x = c_x = 0 \quad (x = 0, a_1;\ 0 \leqslant y \leqslant a_2;\ 0 \leqslant z \leqslant 1), \tag{5.1}$$

$$v = w_y = u_y = \theta_y = c_y = 0 \quad (y = 0, a_2;\ 0 \leqslant x \leqslant a_1;\ 0 \leqslant z \leqslant 1), \tag{5.2}$$

and we seek separable solutions of the form

$$(w, \theta, c) = (W(z), \Theta(z), C(z)) \cos\frac{m_1 \pi x}{a_1} \cos\frac{m_2 \pi y}{a_2}, \tag{5.3}$$

with similar definitions for u and v. m_1 and m_2 run over all non-negative integers.

The equations governing the onset of stationary convection are still (3.1) and (2.2) but with α replaced by λ, where λ is an effective wavenumber given by

$$\lambda^2 = \left[\left(\frac{m_1}{a_1}\right)^2 + \left(\frac{m_2}{a_2}\right)^2\right]\pi^2. \tag{5.4}$$

In figure 4, B_T is given as a function of a_1 for $a_2 = 0.5$ and $B_\mathrm{S} = 100$. Also we have fixed $L = K = 10$ and assumed the lower plate to be conducting and impermeable. It may be seen that the preferred mode (i.e. the mode having the lowest critical Marangoni number) is dependent on the box size a_1. In the situation described by

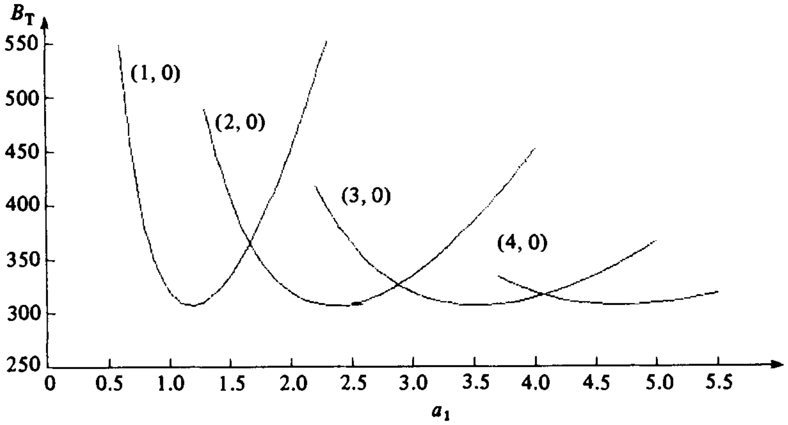

FIGURE 4. Stability curves B_T versus a_1 for $L = K = 10$, $B_S = 100$ at $a_2 = 0.5$. The pairs (m_1, m_2) denote integral number of cycles in (a_1, a_2).

figure 4 this sequence of modes is $m_2 = 0$, $m_1 = 1, 2, 3, 4$. For box sizes $a_1 \sim m_1 \pi / a_2$ with $m_1 = 1, 2, 3, \ldots$ B_T is a minimum at the value 306.5 appropriate to infinite layers. Away from these values the sidewalls exert a stabilizing influence.

6. The onset of oscillatory convective instability

It is the purpose of this section to examine whether oscillatory convective instability, driven by surface-tension forces, occurs. For the situation in which only a temperature gradient is present, Vidal & Acrivos (1966) have shown that oscillatory instability does not occur. The analysis here of the situation where a concentration gradient is also present is similar to that of Vidal & Acrivos and McConaghy & Finlayson (1969); the exact solution to the differential equations is substituted into the boundary conditions, leaving a complex number for the eigenvalue, the Marangoni number. Since this number is real (a ratio of physical parameters), solutions can exist only if the imaginary part of the complex number is zero. The parameter space is searched numerically for situations where this is true.

To keep the notation the same as that of Vidal & Acrivos, the change of variable $\sigma = \sigma_1/Pr$ is made. Then (2.1) may be written as

$$\left.\begin{aligned} [\sigma_1 - Pr(\mathrm{D}^2-a^2)]\,(\mathrm{D}^2-a^2)\,W &= 0 \\ [\sigma_1 - (\mathrm{D}^2-a^2)]\,\Theta &= -W \\ [\sigma_1 - \tau(\mathrm{D}^2-a^2)]\,C = -\tau W, \quad \text{where} \quad \tau &= \frac{Pr}{Sc}. \end{aligned}\right\} \tag{6.1}$$

In order to reduce the amount of numerical work, these will be solved subject to the following more simple but still representative boundary conditions:

$$\left.\begin{aligned} W(0) = \mathrm{D}W(0) = W(1) &= 0, \\ \Theta(0) = \mathrm{D}\Theta(1) &= 0, \\ C(0) = \mathrm{D}C(1) &= 0, \\ \mathrm{D}^2 W(1) &= a^2 B_T\,\Theta(1) + a^2 B_S\,C(1). \end{aligned}\right\} \tag{6.2}$$

For a non-stationary neutral state $\sigma_1 = is$ with s real, in which case (6.1) and (6.2) may be solved analytically to obtain the relationship

$$a_3 + \mathrm{i}a_4 = B_T(a_1 + \mathrm{i}a_2) + \tau B_S(a_5 + \mathrm{i}a_6), \tag{6.3}$$

where $a_1, a_2, \ldots, a_6$ are real-valued analytic functions of α, s, Pr and Sc, which may be obtained from the author on request. When $B_S = 0$, (6.3) reduces to the equation obtained by Vidal & Acrivos when no concentration gradient is present.

Solving for B_T, one obtains

$$B_T = \frac{a_1(a_3 - \tau B_S a_5) + a_2(a_4 - \tau B_S a_6)}{a_1^2 + a_2^2} + i\frac{a_1(a_4 - \tau B_S a_6) - a_2(a_3 - \tau B_S a_5)}{a_1^2 + a_2^2}.$$

Since B_T must clearly be real

$$a_1(a_4 - \tau B_S a_6) - a_2(a_3 - \tau B_S a_5) = 0. \tag{6.4}$$

For fixed values of B_S it must now be determined which values of α, Pr, Sc and s satisfy (6.4). If the principle of the exchange of stabilities is valid, only the value $s = 0$ will make $\operatorname{Im} B_T$ vanish.

7. Results (1)

At first only non-negative values of both B_T and B_S were considered. In particular, B_S was given the values 0, 10, 20, ..., 70, 75. (B_T becomes negative at values of B_S slightly above 75, the exact value depending on the Prandtl and Schmidt numbers.) With $\tau = Pr/Sc$, a value of Pr was fixed and numerical calculations were performed for $\tau^{-1} = 0.01, 0.5, 0.9, 5.0, 10^2$ and 10^3. The values used for Pr were 0.01, 0.5, 0.9, 5.0, 10^2 and 10^3, together with the two asymptotic conditions $Pr \to 0$ and $Pr \to \infty$. α was varied from 0 to 10 in steps of 0.5.

In all cases it was found that the only real value of s that satisfied

$$a_1(a_4 - \tau B_S a_6) - a_2(a_3 - \tau B_S a_5) = 0$$

was $s = 0$. This indicates that for B_S in the range (0, 75) the marginal or neutral state for the surface-tension-driven convection is stationary.

This result is in agreement with previous work. When B_T and B_S are both positive then both temperature and concentration effects are destabilizing. This was precisely the situation for which Joseph & Shir (1968), in their analysis of buoyancy-driven convection, concluded that the principle of the exchange of stabilities is valid. However, it should be pointed out that in their case, in order to have both effects destabilizing, they had to heat from below and salt from above. Yet, even if one both heats and salts from below in the case of surface-tension-driven convection it is possible that the principle of the exchange of stabilities is valid.

This could occur if both the rate of change of surface tension with temperature and with concentration were negative. Almost all materials satisfy the former; most water-soluble organic compounds would satisfy the latter (see Schwartz & Perry 1966).

8. Results (2)

In this section negative values of B_S are considered. If oscillatory instability occurs, the neutral stability locus will be characterized by non-zero frequency factors s which make $\operatorname{Im} B_T$ vanish for a given Prandtl number, Schmidt number and wavenumber.

For many of the fluids with which the crystal grower works (e.g. liquid metals, semiconductors and oxides) tabulated values of the effect of increasing concentration on the surface tension of the fluid are evidently scarce. Thus, at present, it is not known whether for a given positive concentration gradient B_S will be negative or

Nature of instability	Schmidt number Sc	Critical frequency factor s^c	Critical Marangoni number B_T^c	Critical wavenumber α^c
Stationary	All	0	129.607	1.99
Oscillatory	50	−0.70	85.472	1.66
Oscillatory	20	−0.42	116.44	1.45
Oscillatory	10	−0.43	124.94	1.92
Oscillatory	8	−0.21	129.014	1.90
Oscillatory	7.85	−0.21	129.020	1.93
Oscillatory	7.80	−0.14	129.634	1.87

TABLE 3

positive. However, this is not the case when growing crystals from aqueous solution; for most inorganic salts in aqueous solution the rate of change of surface tension with concentration is positive.

We consider now the specific example of a 4 % solution of $MgSO_4$ in water. From Weast (1982) the Prandtl and Schmidt numbers may be calculated and are $Pr = 2.0$ and $Sc = 1700$ (correct to 2 significant figures). When both temperature and concentration gradients are positive we find that $B_T > 0$ and $B_S < 0$.

To consider a particular case, $B_S = -50$ was taken. It was found that oscillatory instability can occur, and in table 3 the value of the critical Marangoni number for the onset of oscillatory instability is recorded and compared to the value obtained in the case of stationary convection ($s = 0$).

For the calculations carried out in table 3, Pr was set equal to 2. It was found that a Schmidt number Sc^* exists such that for all $Sc < Sc^*$ the critical Marangoni number for oscillatory instability is always greater than the critical Marangoni number for stationary instability. Hence for $Sc < Sc^*$ convection will always occur as stationary convection, and it was found numerically that $Sc^* = 7.80$.

Hence it is clear that for the solution of $MgSO_4$ under consideration oscillatory convection will be preferred. However, for the example cited by Hurle & Jakeman (1971) of a water–methanol system we may conclude that for positive temperature and concentration gradients overstability is ruled out. In that case $Pr = 20$ and $Sc = 890$ (for a 4 % solution), although $B_T > 0$ and $B_S > 0$, and the results of §7 then show that stationary convection is preferred.

Again the results in this section are in agreement with previous writers. McConaghy & Finlayson (1969) found that when two effects were opposing (in their case the effect of temperature was destabilizing and the effect of rotation was stabilizing) then oscillatory instability occurs and is preferred for a certain range of Prandtl number.

Finally we note that in space gravity is never zero but very small. In this situation we may regard B_T and B_S as bounded perturbations to a symmetric operator (provided we heat from below and salt from above). For sufficiently small B_T and B_S it may then be deduced from Davis' (1969*b*) results that, in contrast with the zero-gravity case, the possibility of overstability is excluded, regardless of the sign of B_T and B_S. Such a situation could occur when the fluid layer is very shallow.

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